Solutions for Rumack's Preparation Workbook: 4.2

1. Let N be the smaller number. If the sum of two numbers is 27, $N + 15 = 27$. Rearrange the equation and write a subtraction statement to solve for N. $N = 27 - 15 = 12$. The answer is (D).

2. Let N be the smaller number and M be the larger number. The difference between the two numbers is 18, so $M - N = 18$. If the smaller number is half the difference, then $\frac{1}{2} \times 18 = 9$. That means $M - 9 = 18$. Rearrange the equation to solve for M by writing an addition statement. $M = 18 + 9 = 27$. The answer is (B).

3. Let N be the smaller number. If the larger number is triple the smaller number, then the larger number can be represented by $3 \times N = 3N$. Since both numbers have a sum of 180, $N + 3N = 180, 4N = 180, \frac{4N}{4} = \frac{180}{4}, N = 45$. The answer is (C).

4. Let M represent the larger number. If the difference between the two numbers is 64, and the smaller number is 32, then $M - 32 = 64$. Rearrange the equation to solve for M. $M = 64 + 32 = 96$.

5. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 17$ and $M = 2N - 1$. Substitute the 2nd equation into the first by replacing M in the first equation with $2N - 1$. $2N - 1 + N = 17$, $3N - 1 = 17$, $3N - 1 + 1 = 17 + 1$, $3N = 18$, $N = \frac{18}{3} = 6$. The answer is (D).

6. Let M represent the larger number and N represent the smaller number. Set up the equations: $M - N = 77$ and $M = 4N + 2$. Substitute the 2nd equation into the first by replacing M in the first equation with $4N + 2$. $4N + 2 - N = 77$, $3N + 2 = 77, 3N + 2 - 2 = 77 - 2, 3N = 75, N = \frac{75}{3} = 25$. The answer is (E).

7. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 477$ and $M = 3N - 3$. Substitute the 2nd equation into the first by replacing M in the first equation with $3N - 3$. $3N - 3 + N = 477$, $4N - 3 = 477, 4N - 3 + 3 = 477 + 3$, $4N = 480$, $N = \frac{480}{4} = 120$. The answer is (C).

8. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 28$ and $2M + 3N = 64$. Rearrange the first equation so that one of the variables is isolated. For example, $M = 28 - N$. Substitute the rearranged equation into the second equation, replacing M with $28 - N$. $2(28 - N) + 3N = 64$. Now that there is one variable, solve for N. $2 \times 28 + 2 \times (-N) + 3N = 64$, $56 - 2N + 3N = 64$, $56 + N = 64$, $N = 64 - 56 = 8$. If $M + N = 28$, then $M + 8 = 28$, $M = 28 - 8 = 20$. The answer is (D).

9. Let M represent the larger number and N represent the smaller number. Set up the equations: $M - N = 10$ and $4N - M = 53$. Rearrange the first equation so that one of the variables is isolated. For example, $M = N + 10$. Substitute the rearranged equation into the 2nd equation so that M is replaced by $N + 10$. $4N - (N + 10) = 53$. Now that there is one variable, solve for N. $4N - N - 10 = 53$, $3N - 10 = 53, 3N - 10 + 10 = 53 + 10, 3N = 63, N = \frac{63}{3} = 21$. The answer is (B).

10. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 101$ and $M = N + 1$. Substitute the 2nd equation into the 1st, replacing M with $N + 1$. $N + 1 + N = 101, 2N + 1 = 101, 2N + 1 - 1 = 101 - 1, 2N = 100, N = 50$. The answer is (D).

11. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 27$ and $M = 2N$. Substitute the 2nd equation into the first, so that M is replaced by $2N$. $2N + N = 27$, $3N = 27$, $N = \frac{27}{3} = 9$. The answer is (B).

12. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 204$ and $N = M - 12$. Substitute the 2nd equation into the 1st, replacing N with $M - 12$. $M + (M - 12) = 204$, $2M - 12 = 204$, $2M - 12 + 12 = 204 + 12$, $2M = 216$, $M = \frac{216}{2} = 108$. The answer is (B).

13. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 52$ and $M = 3N$. Substitute the 2nd equation into the 1st, replacing M with $3N$. $3N + N = 52$, $4N = 52, N = \frac{52}{4} = 13$. The larger number is $3 \times 13 = 39$. The answer is (B).

14. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 58$ and $M - 4 = N$. Substitute the 2nd equation into the 1st, replacing N with $M - 4$. $M + (M - 4) = 58$, $M + M - 4 = 58, 2M - 4 = 58, 2M - 4 + 4 = 58 + 4, 2M = 62, M = \frac{62}{2} = 31$. The smaller number is $31 - 4 = 27$. The answer is (C).

15. Let M represent the larger number and N represent the smaller number. Set up the equations: $M - N = 49$ and $N = \frac{1}{2}M + 1$. Substitute the 2nd equation into the 1st, replacing N with $\frac{1}{2}M + 1$. $M - (\frac{1}{2}M + 1) = 49$, $M - \frac{1}{2}M - 1 = 49, 1M - \frac{1}{2}M - 1 = 49, \frac{2}{2}M - \frac{1}{2}M - 1 = 49, \frac{1}{2}M - 1 = 49, \frac{1}{2}M - 1 + 1 = 49 + 1, \frac{1}{2}M = 50, \frac{M}{2} = 50, M = 50 \times 2 = 100$. The smaller number is $\frac{1}{2}(100) + 1 = 50 + 1 = 51$. The answer is (C).

16. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 17$ and $2M + 3N = 39$. Rearrange the 1st equation so that one of the variables is isolated. $M = 17 - N$. Substitute the rearranged equation into $2M + 3N = 39$, replacing M with $17 - N$. $2(17 - N) + 3N = 39$. $34 - 2N + 3N = 39, 34 + N = 39, N = 39 - 34 = 5$. The larger number is $17 - 5 = 12$. The answer is (E).

17. Let M represent the larger number and N represent the smaller number. Set up the equations: $M + N = 201$ and $2N + \frac{1}{2}M = 177$. Rearrange the first equation so that the variable N is isolated. $N = 201 - M$. Substitute the rearranged equation into $2N + \frac{1}{2}M = 177$, replacing N with $201 - M$. $2(201 - M) + \frac{1}{2}M = 177$. $402 - 2M + \frac{1}{2}M = 177$. $402 - \frac{4}{2}M + \frac{1}{2}M = 177$, $402 - \frac{3}{2}M = 177$, $402 - \frac{3}{2}M - 402 = 177 - 402, -\frac{3}{2}M = -225$. $\left(-\frac{3}{2}M\right)(2) = -225(2), -3M = -450, \frac{-3M}{-3} = \frac{-450}{-3}$, $M = 150$. The answer is (D).

18. Let M represent the larger number and N represent the smaller number. Set up the equations: $M - N = 101$ and $2M - 3N = 200$. Rearrange the first equation so that the variable M is isolated.

$M = 101 + N$. Substitute the rearranged equation into $2M - 3N = 200$. $2(101 + N) - 3N = 200$. Expand and solve for N: $202 + 2N - 3N = 200$, $202 - N = 200$, $202 - N - 202 = 200 - 202$, $-N = -2$, $\frac{-N}{-1} = \frac{-2}{-1}$, $N = 2$. The answer is (D).

19. Let M and N be the two integers. Set up the equations: $2M + N = 127$ and $2N + M = 104$. Rearrange one equation so that one of the variables is isolated. $N = 127 - 2M$. Substitute the rearranged equation into the other equation $2N + M = 104$: $2(127 - 2M) + M = 104$. Solve for M: $254 - 4M + M = 104$, $254 - 4M + 1M - 254 = 104 - 254$, $-3M = -150$, $\frac{-3}{-3}M = \frac{-150}{-3}$, $M = 50$. Substitute $M = 50$ into one of the equations to solve for N: $N = 127 - 2(50)$, $N = 127 - 100$, $N = 27$. Finally, find the difference between the 2 numbers. $M - N = 50 - 27 = 23$. The answer is (C).

20. Let M be the larger number and N be the smaller number. Set up the equations: $M - N = 6$. $3M + 3N = 30$. Rearrange the first equation so that one of the variables is isolated. $M = 6 + N$. Substitute the rearranged equation into the second equation: $3(6 + N) + 3N = 30$. Solve for N: $18 + 3N + 3N = 30$, $18 + 6N = 30$, $18 + 6N - 18 = 30 - 18$, $6N = 12$, $\frac{6N}{6} = \frac{12}{6}$, $N = 2$. Substitute $N = 2$ into one of the equations to solve for N: $M = 6 + N$, $M = 6 + 2 = 8$. The answer is (B) 8, 2.

21. Set up the equations: $M + N = 210$ and $5N - M = 30$. Rearrange the first equation so that the variable N is isolated. $N = 210 - M$. Substitute the rearranged equation into the second equation $5N - M = 30$: $5(210 - M) - M = 30$. Solve for M: $1050 - 5M - 5M = 30$, $1050 - 10M = 30$, $1050 - 10M - 1050 = 30 - 1050$, $-10M = -1020$, $\frac{-10M}{-10} = \frac{-1020}{-10}$, $M = 102$.

22. Set up the equations: $M + N = 91$ and $3M + 2N = 226$. Rearrange the first equation so that the variable M is isolated: $M = 91 - N$. Substitute the rearranged equation into the second equation $3M + 2M = 226$: $3(91 - N) + 2N = 226$, $273 - 3N + 2M = 226$, $273 - N = 226$, $273 - N - 273 = 226 - 273$, $-N = -47$, $\frac{-N}{-1} = \frac{-47}{-1}$, $N = 47$. The answer is (C).

23. Set up the equations: $2(X + Y) = 320$ and $2X + \frac{1}{2}Y = 230$. Pick one of the equations and rearrange it so that one of the variables is isolated. For example, expand the first equation and then rearrange so that X is isolated: $2(X + Y) = 320$, $2X + 2Y = 320$, $2X + 2Y - 2Y = 320 - 2Y$, $2X = 320 - 2Y$, $\frac{2X}{2} = \frac{320-2Y}{2}$, $X = \frac{320}{2} - \frac{2Y}{2}$, $X = 160 - Y$. Substitute the rearranged equation into the equation $2X + \frac{1}{2}Y = 230$. $2(160 - Y) + \frac{1}{2}Y = 230$. Expand and solve for Y. $320 - 2Y + \frac{1}{2}Y = 230$. $320 - \frac{4}{2}Y + \frac{1}{2}Y = 230$, $320 - \frac{3}{2}Y - 320 = 230 - 320$, $\frac{-3}{2}Y = -90$, $(2)\frac{-3}{2}Y = -90(2)$, $-3Y = -180$, $\frac{-3Y}{-3} = \frac{-180}{-3}$, $Y = 60$.Substitute $Y = 60$ into $X = 160 - Y$ and solve for X: $X = 160 - 60 = 100$. The answer is (A).

24. The equations are already given: $X + Y = 100$ and $X - Y = 50$. Rearrange one of the equations so that one of the variables is isolated. For example, rearrange the second equation so that the variable X is isolated: $X = 50 + Y$. Substitute the rearranged equation into the first equation $X + Y = 100$: $(50 + Y) + Y = 100$. Remove the brackets and solve for Y: $50 + 2Y = 100$, $50 + 2Y - 50 = 100 - 50$, $2Y = 50$, $\frac{2Y}{2} = \frac{50}{2}$, $Y = 25$. Substitute $Y = 25$ into one of the equations to solve for X: $X = 50 + 25 = 75$. The answer is (B) 75, 25.

25. The equations are already given: $X + Y = 0$ and $X - Y = -20$. Rearrange one of the equations so that one of the variables is isolated. For example, rearrange the second equation so that the variable X is isolated: $X = -20 + Y$. Substitute the rearranged equation into the first equation $X + Y = 0$: $(-20 + Y) + Y = 0$. Remove the brackets and solve for Y: $-20 + 2Y = 0, -20 + 2Y + 20 = 0 + 20,$ $2Y = 20, \frac{2Y}{2} = \frac{20}{2}, Y = 10$. Substitute $Y = 10$ into one of the equations and solve for X: $X = -20 + 10 = -10$. The answer is (D) -10, 10.