Solutions for Rumack's Preparation Workbook: 4.3

1. Let n represent the largest number. Set up the equation: $n+(n-2)+(n-4)+(n-6)=244$. Solve for n: $n+n+n+n-2-4-6=244$, $4n-12=244$, $4n-12+12=244+12$, $4n=256$, $\frac{4n}{4}=\frac{256}{4}$, $n=64$. The answer is (C).

2. Let n represent the smallest number. $n+(n+1)+(n+2)+(n+3)+(n+4)=65$. Solve for n: $n+n+n+n+n+1+2+3+4=65$, $5n+10=65$, $5n+10-10=65-10$, $5n=55$, $\frac{5n}{5}=\frac{55}{5}$, $n=11$. The first number is 11. The answer is (D).

3. Let n represent the smallest number. $n+(n+3)+(n+6)=306$. Solve for n: $n+n+n+3+6=306$, $3n+9=306$, $3n+9-9=306-9$, $3n=297$, $\frac{3n}{3}=\frac{297}{3}$, $n=99$. The middle number is $n+3=99+3=102$. The answer is (C).

4. Let n represent the number of points that Jen scored. Then $n+5$ represents the number of points that Keisha scored. Nazira scored $n+5+12=n+17$ points. The sum of their scores is 52: $n+(n+5)+(n+17)=52$. Solve for n: $3n+5+17=52$, $3n+22=52$, $3n+22-22=52-22$, $3n=30$, $\frac{3n}{3}=\frac{30}{3}$, $n=10$. Nazira scored $n+17=10+17=27$ points. The answer is (A).

5. First, find Erica's average of her first three test marks. $\frac{77+81+85}{3}=\frac{243}{3}=81$. Next, find her average after taking four tests: $\frac{243+89}{4}=\frac{332}{4}=83$. Her average improved by $83-81=2$ marks. The answer is (C).

6. First, set up an equation. Let n represent the smallest number. $n+(n+2)+(n+4)+(n+6)+(n+8)+(n+10)+(n+12)+(n+14)+(n+16)+(n+18)=100$. Solve for n: $10n+90=100$, $10n+90-90=100-90$, $10n=10$, $\frac{10n}{10}=\frac{10}{10}$, $n=1$. Next, find the average of all the numbers. Since the sum is already given as 100, the average is 100 ÷ 10 = 10.The difference between the average number and the smallest number is $10-1=9$. The answer is (D).

7. First, set up an equation. Let n represent the smallest number. Since consecutive odd multiples of five increase by 10, $n+(n+10)+(n+20)=135$. Remove the brackets and solve for n. $3n+30=135$, $3n+30-30=135-30$, $3n=105$, $\frac{3n}{3}=\frac{105}{3}$, $n=35$. The answer is (D).

8. First set up an equation. Let n represent the smallest number. Since consecutive even numbers increase by 2, $n+(n+2)+(n+4)+(n+6)+(n+8)=40$. Remove the brackets and solve for n. $5n+20=40$, $5n+20-20=40-20$, $5n=20$. $\frac{5n}{5}=\frac{20}{5}$, $n=4$. The numbers are 4, 6, 8, 10 and 12. The next consecutive even number is 14, and the average of the six numbers is $\frac{4+6+8+10+12+14}{6}=\frac{54}{6}=9$. The answer is (B).

9. Since the answer is in a percentage, it's not necessary to change the percentages. The sum of the four test scores is $76+78+80+82=316$. To raise the average to 80% after taking five tests, find the sum of five tests with a score of 80. $80\times 5=400$. Subtract 316 from 400 to get the percentage the student needs to get on the fifth test.$400-316=84$. The answer is (B).

10. To find the average when the sum is given, it is not necessary determine what each number is. Simply divide the sum by the number of numbers: 296 ÷ 4 = 74. The answer is (D).

11. Set up an equation. Let n represent the smallest number. Consecutive numbers increase by one. $n + (n + 1) + (n + 2) = 123$. Remove the brackets and solve for n. $3n + 3 = 123$, $3n + 3 - 3 = 123 - 3$, $3n = 120$, $\frac{3n}{3} = \frac{120}{3}$, $n = 40$. The answer is (D).

12. Set up an equation. Let n represent the smallest number. Consecutive multiples of 2 increase by two. $n + (n + 2) + (n + 4) + (n + 6) = 404$. Remove the brackets and solve for n. $4n + 12 = 404$, $4n + 12 - 12 = 404 - 12$, $4n = 392$, $\frac{4n}{4} = \frac{392}{4}$, $n = 98$. The answer is (A).

13. Set up an equation. Let n represent the largest number. $n + (n - 1) + (n - 2) + (n - 3) + (n - 4) + (n - 5) = 201$. Remove the brackets and solve for n. $6n - 15 = 201$, $6n - 15 + 15 = 201 + 15$, $6n = 216$, $\frac{6n}{6} = \frac{216}{6}$, $n = 36$. The answer is (D).

14. To find the average, find the sum of the 5 numbers and divide by five. $\frac{284+81}{5} = \frac{365}{5} = 73$. The answer is (B).

15. To find the average, find the sum of the four numbers and divide by four. $930 + 330 = 1260$. $1260 \div 4 = 315$. The answer is (B).

16. Set up an equation for the average of the six tests. $\frac{62 + sum\ of\ scores\ on\ best\ five\ tests}{6} = 77$. Find the sum of the scores on the best five tests by rearranging the equation so that the sum of scores on the best five tests is isolated. $62 + sum\ of\ scores\ on\ best\ five\ tests = 77 \times 6$, $62 + sum\ of\ scores\ on\ best\ five\ tests = 462$, $sum\ of\ scores\ on\ best\ five\ tests = 462 - 62 = 400$. Finally, find the average of the best five tests: $Average\ of\ best\ five\ tests = \frac{400}{5} = 80$.

17. First, find the total number of goals needed to be scored in the playoffs to average 4 a game: $4 \times 4 = 16$. Next, find the total score over the 3 games that have been played so far in the play-offs: $3 \times 2 = 6$. Finally, find the difference between 16 and 6. $16 - 6 = 10$. The answer is (D).

18. The average height of the plants that grew can be calculated by dividing the total heights by the number of plants that grew. First, find the sum of the heights of the 6 plants that grew last season. Each answer is provided in the unit of inches, so convert each measurement to inches. $6 \times 1\ foot\ 8\ inches = 6 \times (12\ inches + 8\ inches) = 6 \times 20\ inches = 120\ inches$. Next, find the sum of the heights of the 9 plants that grew this year. $9 \times 2\ feet\ 1\ inch = 9 \times (24\ inches + 1\ inch) = 9 \times 25\ inches = 225\ inches$. Then, add the two totals together, and divide by the number of plants that grew: $\frac{120+225}{6+9} = \frac{345}{15} = 23$. The answer is (B).

19. Set up an equation. Let n represent the largest number. $n + (n - 1) + (n - 2) + (n - 3) + (n - 4) + (n - 5) + (n - 6) + (n - 7) + (n - 8) + (n - 9) = 415$. Remove the brackets and solve for n. $10n - 45 = 415$, $10n - 45 + 45 = 415 + 45$, $10n = 460$, $\frac{10n}{10} = \frac{460}{10}$, $n = 46$. The answer is (B).

20. Set up an equation. Let n represent the smallest number. $n + (n + 15) + (n + 30) + (n + 45) = 810$. Remove the brackets and solve for n. $4n + 90 = 810$, $4n + 90 - 90 = 810 - 90$, $4n = 720$,

$\frac{4n}{4} = \frac{720}{4}, n = 180$. The largest number is $n + 45 = 180 + 45 = 225$. The difference between first and last numbers of the sequence is $225 - 180 = 45$. The answer is (C).

21. Let n represent the third number. The average of the three numbers is $\frac{580+n}{3} = 285$.Rearrange the equation to solve for n. $580 + n = 285 \times 3$, $580 + n = 855$, $580 + n - 580 = 855 - 580$, $n = 275$. The answer is (D).

22. The combined average is calculated by adding the two sums together and dividing by the size: $96 + 156 = 252$. There are 7 numbers, so divide the total by 7. $252 \div 7 = 36$. The answer is (C).

23. Let n represent the total number of eyes drawn by the other two children. The average number of eyes is $\frac{n+12}{3} = 7$. Rearrange the equation to solve for n. $n + 12 = 7 \times 3$, $n + 12 = 21$, $n = 21 - 12 = 9$. The total number of eyes drawn by the other two children is 9, so the answer choice must have two numbers that have a sum of 9. The anwer is (C) 3, 6.

24. Let n represent the smallest number. Set up an equation: $n + (n + 7) + (n + 14) + (n + 21) + (n + 28) = 245$. Remove the brackets and solve for n. $5n + 70 = 245$, $5n + 70 - 70 = 245 - 70$, $5n = 175$, $\frac{5n}{5} = \frac{175}{5}$, $n = 35$. The middle number is $n + 14 = 35 + 14 = 49$. The answer is (C).

25. For each essay, multiply the number of words by the average length of each word. $1000 \times 4.1 = 4100$ and $1500 \times 4.6 = 6900$. Calculate the total number of letters in both essays by finding the sum of 4100 and 6900. $4100 + 6900 = 11{,}000$. Divide this sum by the total number of words in both essays. $11{,}000 \div (1000 + 1500) = 11{,}000 \div 2500 = 4.4$. The answer is (B).