Solutions for Rumack's Preparation Workbook: 4.4

1. Let W represent the width and $W+4$ represent the length. The formula for the perimeter of a rectangle is $P=2L+2W$. Since the perimeter is 48 units, $48=2L+2W$. Replace L with the expression for length $W+4$: $48=2(W+4)+2W$. Solve for W: $48=2W+8+2W$, $48=4W+8$, $48-8=4W+8-8$, $40=4W$, $\frac{40}{4}=\frac{4w}{4}$, $10=W$. The length is $W+4=10+4=14$. The answer is (C).

2. Let b represent the length of the smaller base, $2b$ represent the length of the larger base, and $\frac{1}{2}b$ represent the lengths of the other sides. Since the perimeter is 88 units, $P=b+2b+\frac{1}{2}b+\frac{1}{2}b$. $88=3b+b$, $88=4b$, $\frac{88}{4}=\frac{4b}{4}$, $22=b$. The answer is (C).

3. Let b represent the base and $2b$ represent the height. The formula for the area of a triangle is $A=b\times h\div 2$. Since area is given, and height is $2b$, $121=b\times(2b)\div 2$, $121=2b^2\div 2$, $121\ \times 2= 2b^2\div 2\ \times 2$, $242=2b^2$, $242\ \div 2=2b^2\ \div 2$, $121=b^2$, $\sqrt{121}=\sqrt{b^2}$, $11=b$. The height is $2b=2(11)=22\ mm$. The answer is (A).

4. Let w represent the width and $3w$ represent the length. The equation for perimeter of a rectangle is $P=2L+2W$, $120=2(3W)+2W$, $120=6W+2W$, $120=8W$, $\frac{120}{8}=\frac{8W}{8}$, $15=W$. The length is $3W=3(15)=45\ cm$. The answer is (B).

5. The volume of the first prism is $V_1=L_1\times W_1\times H_1=5\ cm\times 4\ cm\times 8\ cm=160\ cm$. The volume of the second prism is $V_2=L_2\times W_2\times H_2$, $160=2(5)\times W_2\times\frac{1}{4}(8)$, $160=10\times W_2\times\frac{8}{4}$, $160=\ 20\ \times W_2$, $160\ \div 20=20\times\ W_2\ \div 20$, $80=W_2$.The answer is (C).

6. Let n represent the shortest side length. The other side lengths are $n+4$ and $n+8$. The perimeter is $48=n+(n+4)+(n+8)$. Solve for n. $48=3n+12$. $48-12=3n+12-12$, $36=3n$, $\frac{36}{3}=\frac{3n}{3}$, $12=n$. Since it is a right triangle, the hypotenuse is the longest side length, and the other two sides can represent the base and height. The two sides are 12 units and $12+4=16$ units. Since the triangle is a right angle, one of the sides will be the base and the other will be the height. Substitute the base and height into the equation $A=b\ \times h\ \div 2$: $A=12\ \times 16\ \div 2.$, $A=192\ \div 2$, $A=96$. The answer is (B).

7. Let W represent the width and $W+11$ represent the length. The perimeter is $P=2L+2W$, $46=2(W+11)+2W$. $46=2W+22+2W$, $46=4W+22$, $46-22=4W+22-22$, $24=4W$, $\frac{24}{4}=\frac{4W}{4}$, $6=W$. The answer is (D).

8. Let W represent the width and $W+1$ represent the length. The perimeter is $P=2L+2W$, $198\ mm=2(W+1)+2W$, $198=2W+2+2W$, $198=4W+2$, $198-2=4W+2-2$, $196=4W$, $\frac{196}{4}=\frac{4W}{4}$, $49=W$. The answer is (C).

9. The volume of the first prism is $V_1=L_1\times W_1\times H_1=11\times 6\times 5=330\ units^3$. The volume of the second prism is $V_2=L_2\times W_2\times H_2=11\times W_2\times 2(5)=330\times\frac{1}{3}$. Solve for W_2. $110\ \times W_2=110$, $W_2=1$ unit. The answer is (D).

10. Let C represent the length of the common side. The length of one rectangle is $C + 5$. The length of the other rectangle is $C + 2$. If the length around the outside is $56\, units^2$, then $56 = 3C + 2(C + 5) + 2(C + 2), 56 = 3C + 2C + 10 + 2C + 4, 56 = 7C + 14, 56 - 14 = 7C + 14 - 14, 42 = 7C$, $\frac{42}{7} = \frac{7C}{C}, 6 = C$.

11. Let W represent the width and $4W$ represent the length. The perimeter is $P = 2L + 2W$, $50 = 2(4W) + 2W, 50 = 8W + 2W, 50 = 10W, \frac{50}{10} = \frac{10W}{10}, 5 = W$. The length is $4W = 4(5) = 20\, units$. The area is $A = L \times W = 20 \times 5 = 100\, units^2$.

12. Let the longest side length be n. The perimeter is $P = n + (n - 4) + (n - 8) + (n - 12)$. $40 = 4n - 24$, $40 + 24 = 4n - 24 + 24$, $64 = 4n$, $\frac{64}{4} = \frac{4n}{4}$, $16 = n$. The answer is (C).

13. Let h represent the height and the $\frac{1}{2}h$ represent the base. If the area is $A = b \times h \div 2$, $36 = (\frac{1}{2}h)\times h \div 2$, $36 = \frac{1}{2}h^2 \times \frac{1}{2}$, $36 = \frac{h^2}{4}$, $(4)36 = \frac{h^2}{4}(4)$, $144 = h^2, \sqrt{144} = \sqrt{h^2},\ 12 = h$. The base is $\frac{1}{2}h = \frac{1}{2}(12) = 6\, units$. The answer is (C).

14. If the ratio of the perimeter to the area is 1:1, then the perimeter equals the area. If the base and height are 6 and 8 units respectively, the area is $A = b \times h \div 2 = 6 \times 8 \div 2 = 48 \div 2 = 24\, units^2$. The perimeter is $P = 6 + 8 + hypotenuse$, $24 = 14 + h$, $24 - 14 = 14 + hypotenuse - 14$, $10 = hypotenuse$. The answer is (B).

15. Let L represent the length and W represent the width. If the ratio of length to perimeter is 1:3, then $P = 3L$. If the length is 10 units, then the perimeter is $3(10) = 30\, units$. $P = 2L + 2W$, $30 = 2(10) + 2W$, $30 = 20 + 2W$, $30 - 20 = 20 + 2W - 20$, $10 = 2W, \frac{10}{2} = \frac{2W}{2}, 5 = W$. The answer is (D).

16. Let L represent the largest two sides. The perimeter is $P = L + L + (L - 10) + (L - 20) + (L - 30)$, $140 = 5L - 60$, $140 + 60 = 5L - 60 + 60$, $200 = 5L, \frac{200}{5} = \frac{5L}{5}, 40 = L$. The answer is (D).

17. Let w represent the width. The length is $w + 6$. If the length is 17 cm, $17 = w + 6$, $17 - 6 = w + 6 - 6$, $11 = w$. The perimeter is $P = 2L + 2W = 2(17) + 2(11) = 34 + 22 = 56$. The answer is (E).

18. Let h represent the height and $h + 5$ represent the base. If $h = 7$, then the base is $7 + 5 = 12\, units$. The area is $A = b \times h \div 2 = 12 \times 7 \div 2 = 42\, units^2$. The answer is (B).

19. Let s represent the side length of the square. The area is $A = s^2$ and the perimeter is $P = 4s$. Since the ratio of perimeter to area is 1:1, the area equals the perimeter. $s^2 = 4s$. Check each of the answer choices and substitute them into the equation. The answer is (D) 4 units, since $4^2 = 4(4) = 16$.

20. Let b represent the base. Since the ratio of base to height is 4:1, height is represented by $\frac{1}{4}b$. The area is $Area = b \times h$, $196\, cm^2 = b \times h$, $196 = b \times \frac{1}{4}b$, $196 = \frac{1}{4}b^2$, $196 \times 4 = \frac{b^2}{4} \times 4$, $784 = b^2, \sqrt{784} = \sqrt{b^2}$, $28 = b$. The answer is (C).

21. Let h represent the height. The width is $2h$ and the length is $3h$. Since the length is 15 cm, $15 = 3h$, $\frac{15}{3} = \frac{3h}{3}$, $5 = h$. The width is $2(5) = 10$. The volume is $V = L \times W \times H = 15 \times 10 \times 5 = 750\, cm^3$.

22. Let W represent the width of the rectangle. The base (or length) is $5W$. The perimeter is $P = 2L + 2W = 2(5W) + 2W = 10W + 2W = 12W$. Since the perimeter is $36\ mm$, $36 = 12W$, $\frac{36}{12} = \frac{12W}{12}$, $3 = W$.

23. Let s represent the side length of the square. Since the width of the rectangle is $8\ units$, the length is $8 + 10 = 18\ units$. The area of the rectangle is $A = L \times W = 18 \times 8 = 144\ units^2$. The area of the square is $A = s^2 = 144$. $\sqrt{s^2} = \sqrt{144}$, $s = 12$. The answer is (C).

24. Let n represent the length of the longest side. Then $(n - 5)$ and $(n - 10)$ represent the other sides that are consecutive multiples of 5. The perimeter is $P = 10 + 10 + 10 + n + (n - 5) + (n - 10) = 60\ units$. Solve for n: $3n + 30 - 15 = 60$. $3n + 15 = 60$, $3n + 15 - 15 = 60 - 15$, $3n = 45$, $\frac{3n}{3} = \frac{45}{3}$, $n = 15$. The answer is (C).

25. Let s represent the side length of the square and base of the triangle. The height of the triangle is represented by $4s$. The area of a square is $A = s^2$, $16 = s^2$, $\sqrt{16} = \sqrt{s^2}$, $4\ mm = s$. The area of the triangle is $A = b \times h \div 2 = 4(4 \times 4) \div 2 = 64 \div 2 = 32$. The ratio of the area of the square to the area of the triangle is $16{:}\,32 = 1{:}\,2$. The answer is (C).